Mathematics
Quarter 1 – Module 3:
Solving Equations
Transformable to Quadratic Equations (Including Rational Algebraic Equations)
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Mathematics
Quarter 1 – Module 3: Solving Equations Transformable to Quadratic Equations (Including Rational Algebraic Equations)
Introductory Message

For the facilitator:

Welcome to the Mathematics Grade 9 Self-Learning Module (SLM) on solving equations transformable to quadratic equations (including rational algebraic equations)!

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

It focuses on the fundamental concepts of quadratic equations and its application. The presentation and examples herein stated are tailored-fit and meticulously selected to ensure learners understanding. Learners should be able to identify the significant characteristics of each concept. An array of solving strategies are then manifested to guide student’s learning.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher
This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners’ progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.
For the learner:

Welcome to the Mathematics Grade 9 Self-Learning Module (SLM) on Solving equations transformable to quadratic equations (including rational algebraic equations)!

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

- **What I Need to Know**: This will give you an idea of the skills or competencies you are expected to learn in the module.

- **What I Know**: This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.

- **What’s In**: This is a brief drill or review to help you link the current lesson with the previous one.

- **What’s New**: In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.

- **What is It**: This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.

- **What’s More**: This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.

- **What I Have Learned**: This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
### What I Can Do
This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.

### Assessment
This is a task which aims to evaluate your level of mastery in achieving the learning competency.

### Additional Activities
In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.

### Answer Key
This contains answers to all activities in the module.

At the end of this module you will also find:

### References
This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer What I Know before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!
What I Need to Know

This module was designed and written with you in mind. It is here to help you master how to solve equations transformable to quadratic equations. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

The module is divided into two lessons, namely:

- Lesson 1 – Solving Quadratic Equations Not Written in Standard Form
- Lesson 2 – Solving Rational Algebraic Equations Transformable to Quadratic Equations

After going through this module, you are expected to:

a. solve equations transformable to quadratic equations using the most appropriate method (extracting square root, factoring, completing the square or using the quadratic formula; and

b. solve rational algebraic equations transformable to quadratic equations.

What I Know

Directions: In this part, you will find out how much you already know in this module. Select the letter that best answers each of the questions. Record the items that you have answered incorrectly and find the right answers as you go through this module.

1. Which among the following is a quadratic equation?
   A. \((x + 1)^2 = x^2 - 1\)
   B. \((x + 1)(x - 2) = x - 3\)
   C. \(x(x + 1)^2 = 0\)
   D. \(2x^2 = 2(x - 1)^2\)

2. Which of the following does not describe a quadratic equation?
   A. A quadratic equation is an equation of degree 2.
   B. A quadratic equation has at most two solutions.
   C. All rational algebraic equations are transformable to quadratic equations.
   D. The standard form of a quadratic equation is \(ax^2 + bx + c = 0\).
3. The following rational algebraic equations are transformable to quadratic equations EXCEPT
   A. \( \frac{1}{x} - \frac{6}{x} = \frac{2}{3} \)  
   B. \( \frac{x}{2} - \frac{4}{x} = 1 \)  
   C. \( \frac{3}{x-1} + \frac{2}{x} = \frac{1}{3} \)  
   D. \( \frac{5}{2x} + \frac{x}{3} = -4 \)

4. What is the most convenient method to solve the quadratic equation \((x + 4)^2 = 15\)?
   A. extracting square root  
   B. factoring  
   C. completing the square  
   D. quadratic formula

5. Which among the methods can be used to solve all types of quadratic equations?
   I. Extracting square root  
   II. Factoring  
   III. Completing the square  
   IV. Quadratic formula
   A. I and II  
   B. IV only  
   C. III and IV  
   D. I, II, III, IV

6. What quadratic equation has roots of -4 and -5?
   A. \((x + 4)(x + 5) = 10\)  
   B. \((x - 4)(x - 5) = 0\)  
   C. \(x^2 - 20 = -9x\)  
   D. \(x^2 + 25 = -9x + 5\)

7. What are the roots of \(2x^2 = 9x\)?
   A. 1, \(\frac{9}{2}\)  
   B. 0, \(-\frac{9}{2}\)  
   C. \(\frac{9}{2}\)  
   D. 1, \(-\frac{9}{2}\)

8. One of the roots of \((y + 2)(y - 1) = 3y\) is \(1 - \sqrt{3}\). What is the other root?
   A. \(1 + \sqrt{3}\)  
   B. \(-1 - \sqrt{3}\)  
   C. \(-1 + \sqrt{3}\)  
   D. \(-\sqrt{3}\)

9. The rational algebraic equation \(\frac{y}{2} - \frac{4}{y} = 1\) when transformed to quadratic equation in standard form is
   A. \(y^2 - 2y - 8 = 0\)  
   B. \(y^2 + 2y + 8 = 0\)  
   C. \(y^2 + 2y - 8 = 0\)  
   D. \(y^2 - 2y + 8 = 0\)

10. What are the roots of the equation in #9?
    A. -4, 2  
    B. 4, -2  
    C. -4, -2  
    D. 4, 2

11. Which of the equations can be solved using factoring technique?
    A. \(\frac{x}{6} = \frac{2}{x+4}\)  
    B. \(\frac{y}{2} + \frac{3}{y} = y\)  
    C. \(x^2 - 5x = 5x + 25\)  
    D. \((a - 3)^2 = 7a\)

12. Which of the following is true about the equation \(\frac{2}{x} + \frac{3}{x+1} = 4\)?
    A. The equation is not a quadratic equation.  
    B. The equation can be solved using factoring.  
    C. The equation has roots 4 and -2.  
    D. The equation when transformed into quadratic equation in standard form is \(4x^2 - x - 2 = 0\)
For items 13-15, refer to the table below.

### 13-15. Solve for the roots of \((x + 2)^2 = 3x + 6\)

<table>
<thead>
<tr>
<th>Solutions</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>(13)(\text{______} = 3x + 6)</td>
<td>Expand the square of a binomial</td>
</tr>
<tr>
<td>(x^2 + 4x - 3x + 4 - 6 = 0)</td>
<td>Write all terms on one side of the equation.</td>
</tr>
<tr>
<td>(14)(\text{__________})</td>
<td>Write in standard form. Combine similar terms.</td>
</tr>
<tr>
<td>((x - 1)(x + 2) = 0)</td>
<td>Use factoring technique.</td>
</tr>
<tr>
<td>((x - 1) = 0), ((x + 2) = 0)</td>
<td></td>
</tr>
<tr>
<td>(x = 1, x = -2)</td>
<td>Solve for x.</td>
</tr>
</tbody>
</table>

13. What is the correct expression when we expand \((x + 2)^2\)?

A. \(x^2 + 2x + 4\)  
B. \(x^2 + 2x + 2 = 0\)  
C. \(x^2 + 4x + 4\)  
D. \(x^2 + 4x + 2 = 0\)

14. How is the equation written in standard form?

A. \(x^2 + x - 2 = 0\)  
B. \(x^2 + x + 10 = 0\)  
C. \(x^2 + 7x - 2 = 0\)  
D. \(x^2 + 7x + 2 = 0\)

15. What is the correct reason for this item?

A. Distributive property  
B. Zero Product property  
C. Commutative Property  
D. Substitution Property
Lesson 1

SOLVING QUADRATIC EQUATIONS NOT WRITTEN IN STANDARD FORM

Begin this lesson by checking your prior knowledge about the different mathematics principles you learned. These knowledge and skills will help you in conceptualizing how the solutions to equations transformable to quadratic equations be determined and how these concepts are applied in real life. The concept of radicals is a pre-requisite to this lesson. You can refer to prepared modules or video lessons about these concepts to help you in performing each of the activities provided.

What’s In

Activity 1. “Can you Factor it Out”
Determine which of the following quadratic equations can be solved using factoring and which cannot. Answer the questions that follow. Write your answers on a separate sheet of paper.

\[
\begin{align*}
    x^2 + 8x + 16 &= 0 \\
    x^2 + 6x - 40 &= 0 \\
    x^2 + 3x - 9 &= 0 \\
    x^2 - 8x + 15 &= 0 \\
    x^2 + 10x + 6 &= 0 \\
    x^2 - 4x + 1 &= 0
\end{align*}
\]

Questions:
1. Did you find it easy identifying quadratic equations that can be solved by factoring? Why?
2. Write these equations in factored form then solve for the roots.

What’s New

Activity 2: Change Me to Your Standard
Rewrite the following equations into standard form \((ax^2 + bx + c = 0)\). Determine all possible methods that will help you find the roots of the given equations.

1. \(x(x + 4) = -3\)
2. \(5x^2 - 12 = 2x^2 + 3\)
3. \((x + 1)^2 + (x - 2)^2 = 10\)
4. \(2(x + 3)^2 - (x + 2)^2 = 16\)
Quadratic equations can be solved using four different methods namely factoring, extracting square roots, completing the square and quadratic formula. Completing the square and quadratic formula works for all types of quadratic equations. However, if factoring or extracting square roots work, these are often the easiest to use. Note that there are equations that are not written in standard form. Express these first in standard form, $ax^2 + bx + c = 0$ before solving.

You already worked on rewriting equations not written in standard form and finding the roots in Activity 1 and 2. Remember, the first step in solving quadratic equations is to rewrite the equation in standard form. Study the following examples and see how the different methods are used.

**Example #1.** Find the roots of $x(x + 4) = -3$

Solution:

\[
\begin{align*}
x(x + 4) &= -3 \\
x^2 + 4x &= -3 & \text{Use distributive property of multiplication} \\
x^2 + 4x + 3 &= 0 & \text{Rewrite the equation in standard form.}
\end{align*}
\]

Now, what method do we use to solve the equation (solving for the equation means finding the solutions or the roots)? Since factoring is most convenient, check first if it works. Are there any factors of 3 that has a sum of 4?

\[
\begin{align*}
(x + 3)(x + 1) &= 0 & \text{Write the equation in factored form} \\
x + 3 &= 0 \quad \text{or} \quad x + 1 &= 0 & \text{Apply Zero product property} \\
x &= -3 & \quad \text{or} \quad x &= -1 & \text{Solve for the values of } x \text{ (APE)}
\end{align*}
\]

Check if the obtained values of $x$ make the equation $x(x + 4) = -3$ true. By substitution:

\[
\begin{align*}
\text{for } x &= -3 \\
x(x + 4) &= -3 \\
-3(-3 + 4) &= -3 \\
-3(1) &= -3 \\
-3 &= -3 \\
\text{for } x &= -1 \\
-1(-1 + 4) &= -3 \\
-1(3) &= -3 \\
-3 &= -3
\end{align*}
\]

Since the obtained values of $x$ make the equation true, therefore the solutions of the equation are $x = -3 \quad \text{or} \quad x = -1$.

**Example #2.** Find the roots of $5x^2 - 12 = 2x^2 + 3$

Solution:

\[
\begin{align*}
5x^2 - 2x^2 - 12 - 3 &= 0 & \text{Write the equation in standard form.} \\
3x^2 - 15 &= 0 & \text{Notice that } b = 0 \text{ in the equation. Express in the form } x^2 = c, \text{ then use extracting square roots} \\
3x^2 &= 15 & \text{Multiplication Property of Equality (MPE)} \\
x^2 &= 5 & \text{Extract the square root of both sides.} \\
x &= \pm \sqrt{5} & \text{Therefore, the roots are: } x = \sqrt{5} \text{ & } x = -\sqrt{5}
\end{align*}
\]

The checking is for you to do.
Example #3. Solve for the equation \((x + 1)^2 + (x - 2)^2 = 10\)

Solution:

\[
\begin{align*}
(x + 1)^2 + (x - 2)^2 &= 10 \\
x^2 + 2x + 1 + x^2 - 4x + 4 &= 10 \\
x^2 + x^2 + 2x - 4x + 1 + 4 - 10 &= 0 \\
2x^2 - 2 - 5 &= 0
\end{align*}
\]

Why?

Can you solve for the roots using factoring? No, since the equation is not factorable. We may use completing the square or quadratic formula.

Let us use quadratic formula.

\[
a = 2, b = -2, c = -5
\]

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-5)}}{2(2)} = \frac{2 \pm \sqrt{4 + 40}}{4}
\]

\[
x = \frac{2 \pm \sqrt{44}}{2} = \frac{2 \pm 2\sqrt{11}}{2}
\]

Therefore, the solutions of the equation \((x + 1)^2 + (x - 2)^2 = 10\) are \(x = \frac{1 + \sqrt{11}}{2}\) and \(x = \frac{1 - \sqrt{11}}{2}\).

One way of checking if the solutions obtained are correct aside from substituting it to the original equation is by solving it using another method. Use completing the square to check.

Example #4. Solve for \(x\) in the equation \(2(x + 3)^2 - (x + 2)^2 = 16\).

Solution:

\[
\begin{align*}
2(x + 3)^2 - (x + 2)^2 &= 16 \\
2(x^2 + 6x + 9) - (x^2 + 4x + 4) &= 16 \\
2x^2 + 12x + 18 - x^2 - 4x - 4 &= 16 \\
x^2 + 8x - 2 &= 0 \\
\end{align*}
\]

Since the equation is not factorable, let us use completing the square this time.

\[
\begin{align*}
x^2 + 8x - 2 &= 0 \\
x^2 + 8x &= 2 \\
x^2 + 8x + 16 &= 2 + 16 \\
(x + 4)^2 &= 18
\end{align*}
\]

Notice that the equation we have arrived is in the form \((x - p)^2 = d\), we can now proceed to the extracting of square roots.

\[
\begin{align*}
x + 4 &= \pm\sqrt{18} \\
x + 4 &= \pm\sqrt{2\cdot9} \\
x + 4 &= \pm3\sqrt{2} \\
x + 4 &= 3\sqrt{2} && x + 4 = -3\sqrt{2} \\
x &= -4 + 3\sqrt{2} && x = -4 - 3\sqrt{2}
\end{align*}
\]

Therefore, the solutions to the quadratic equation are \(x = -4 + 3\sqrt{2}\) and \(x = -4 - 3\sqrt{2}\). Check by solving the equation using quadratic formula.
When solving equations transformable to quadratic equations:

1. Express the equation in standard form by simplifying both sides of the equation if needed.
2. Notice if there are any common factors in the equation, factor it out in order to work with a simplified quadratic equation.
3. Use the following methods to solve for the roots:
   a. Extracting square is most useful when:
      i) the equation can be written in the form \(x^2 = c\), where \(b = 0\).
      ii) The equation is of the form \((x - p)^2 = d\), where \(p, d\) are real numbers.
   b. Factoring is most useful when:
      i) \(c = 0\) in the equation \(ax^2 + bx + c = 0\); and
      ii) the left side of the equation is factorable.
   c. Completing the square or quadratic formula can be used in all types of quadratic equations. Since the processes of both methods are quite complex, I suggest to only use it when extracting square root or factoring is not applicable unless your teacher told you to do so.
4. Check if the solution obtained is correct by substituting it to the original equation or use another method to solve for the equation again.

What’s More

Activity 3. Match With My Roots
Match the quadratic equations in column A to its corresponding roots in column B. Use any method in solving. Show your solutions on a separate sheet of paper.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>((x - 6)^2 = 2x - 5)</td>
<td>a. 5, -1</td>
</tr>
<tr>
<td>(n^2 - 4n = 5)</td>
<td>b. (2 - \sqrt{19}, 2 + \sqrt{19}),</td>
</tr>
<tr>
<td>((x + 3)(x - 2) = x + 10)</td>
<td>c. (7 - 2\sqrt{2}, 7 + 2\sqrt{2}),</td>
</tr>
<tr>
<td>((a + 1)^2 + (a - 8)^2 = 50)</td>
<td>d. 4, -4</td>
</tr>
<tr>
<td>(y(y - 4) = 15)</td>
<td>e. (\frac{7 - \sqrt{19}}{2}, \frac{7 + \sqrt{19}}{2})</td>
</tr>
</tbody>
</table>

What I Have Learned

Activity 4. Sum it Up!
Let us recall what you have learned. Supply the missing word/s to make the statement true.

1. The first step in solving equations transformable to quadratic equations is to express the equation in _______________________.
2. The most convenient method to use in order to solve quadratic equation is ______________________ when \(c = 0\) in the equation \(ax^2 + bx + c = 0\).
3. Using quadratic formula and _______________________ will solve all types of quadratic equations.

4. Extracting square roots is most useful when the equation can be written of the form _______________________ or _______________________.

**What I Can Do**

You may wonder why we keep on studying quadratic equations. In fact, this concept directly and indirectly helps us in some ways. Some problems may look so complicated but thinking about how to come up with the solutions will help us develop our critical thinking which is important in developing our decision-making skill. Real-life quantities such as profits, yields and losses, area, volume and many others can be represented, but not limited to, quadratic equations in order for the analyst to come up with practical decisions. Such decisions are made to make our life comfortable.

To follow physical distancing protocols and to ensure safety during the time of pandemic, online selling becomes a trend nowadays. Many do this, male or female, young or old or even some students. But they do not seem to realize that they are already doing quadratics through analysing their revenues or losses. Are you one of them?

Another example is shown in the problem below.

**Problem:** The Grade IX-Saturn class of one IP school in Kidapawan was asked to create an open rectangular box using any indigenous materials available in the community. The box should have the following dimensions: width = 5 cm, height = (x + 1) cm, length = (x + 3) cm. See figure below (the figure is drawn not to scale). What should be the measurement of the height and the width to make a box with a volume of 240 cm³? Recall: \( V = \text{length} \times \text{width} \times \text{height} \)

![Diagram of a rectangular box](image)

**Answer the following:**

1. Write a mathematical sentence that represents the volume of the box.
2. Simplify the equation by rewriting it to quadratic equation in standard form.
3. Solve the quadratic equation obtained from #2 using any method.
4. What are the dimensions of the box? *(Note: The measure of each dimension is always positive)*
5. If you were to create your own design of the box, what available materials will you use? What do these materials mean in your community?
6. Can you cite some experiences where the concept of quadratic equations are applied in your daily life?
Lesson 2

SOLVING RATIONAL ALGEBRAIC EQUATIONS TRANSFORMABLE TO QUADRATIC EQUATIONS

Begin lesson 2 by assessing your prior knowledge about the different mathematics principles and skills you’ve learned. These knowledge and skills will help you in conceptualizing how the solutions to rational algebraic equations are solved. You will also learn how these concepts are used in daily life. Perform each of the activities. If you find difficulties in accomplishing the tasks, you can refer to the modules you have gone through previously. You can also refer to video lessons about this including topics on radicals if you have any access.

What’s In

Activity 1. “Least among the Greatest”
Determine the least common multiple of the given groups of expressions.

1. 3, 6, x
2. 4, x, 3x
3. x, x – 1, x + 1
4. x² + 5x + 6, x + 3

Question: How did you determine the least common multiple of the expressions in the activity?

What’s New

Activity 2. “SHOW ME”
Show whether the following equations can be written into quadratic equation or not. Explain how you arrived at your answer. Then answer the questions that follow.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \frac{2}{x} - \frac{x}{4} = \frac{1}{2} )</td>
<td></td>
</tr>
<tr>
<td>2. ( \frac{2x}{x+1} = \frac{1}{x-1} - \frac{2}{x+1} )</td>
<td></td>
</tr>
<tr>
<td>3. ( \frac{1}{x-3} + \frac{x}{x-2} = \frac{1}{x^2-5x+6} )</td>
<td></td>
</tr>
</tbody>
</table>

Questions:
1. Which among the problems is/are quadratic equations?
2. What is your basis in identifying if an equation is quadratic or not?
3. Can you find the solutions of each of the equations above? How?
Rational algebraic equations are mathematical sentences that contain rational algebraic expressions. You learned from your Grade 8 Mathematics that a rational algebraic expression is a ratio of two polynomials given that the denominator is not equal to zero. Rational algebraic equations (but not all) are transformable to quadratic equations.

Since we are working with these types of equations, you must have prior knowledge with rational expressions. There are four steps outlined for you to follow in solving rational algebraic equations transformable to quadratic equations.

**Example #1** Solve \( \frac{2}{x} - \frac{x}{4} = \frac{1}{2} \).

Note that the denominator should not be equal to zero since any number when divided by zero is undefined. Therefore, \( x \neq 0 \) in this problem.

<table>
<thead>
<tr>
<th>Step</th>
<th>Remove the denominator by multiplying both sides of the equation by the Least Common Multiple (LCM) of all denominators. What is the LCM in the given equation? Answer: 4x</th>
<th>( \frac{2}{x} \cdot (4x) - \frac{x}{4} \cdot (4x) = \frac{1}{2} \cdot (4x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( 8 - x^2 = 2x )</td>
</tr>
<tr>
<td>Step 2</td>
<td>Is the resulting equation quadratic? If yes, rewrite the quadratic equation in standard form.</td>
<td>( x^2 + 2x - 8 = 0 )</td>
</tr>
<tr>
<td>Step 3</td>
<td>Determine the roots of the equation obtained using any of the methods. Is the equation factorable? Yes, we use factoring method.</td>
<td>( (x + 4)(x - 2) = 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x + 4 = 0 ) or ( x - 2 = 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x = -4 ) or ( x = 2 )</td>
</tr>
<tr>
<td>Step 4</td>
<td>Check if the values obtained make the equation true.</td>
<td>( \frac{2}{x} - \frac{x}{4} = \frac{1}{2} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x = -4 ) for ( x = 2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{2}{-4} ) ( \neq \frac{1}{2} )</td>
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<td></td>
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<td>( \frac{2}{-4} ) ( \neq \frac{1}{2} )</td>
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<td>( \frac{1}{2} )</td>
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</tbody>
</table>

The solutions to the equation \( \frac{2}{x} - \frac{x}{4} = \frac{1}{2} \) are \( x = -4 \) and \( x = 2 \).
Example #2. Find the roots of \( \frac{2x}{x+1} = \frac{1}{x-1} - \frac{2}{x+1} \)

For the equation to be defined, \( x \neq \pm 1 \).

Solution:

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Remove the denominator by multiplying both sides of the equation by the Least Common Multiple (LCM) of all denominators. What is the LCM in the given equation? Answer: ((x + 1)(x - 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{2x}{x+1}(x+1)(x-1) = \frac{1}{x-1}(x+1)(x-1) - \frac{2}{x+1}(x+1)(x-1) )</td>
</tr>
<tr>
<td></td>
<td>( 2x(x-1) = 1(x+1) - 2(x-1) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 2</th>
<th>Rewrite the quadratic equation in standard form.</th>
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<tr>
<td></td>
<td>( 2x^2 - 2x = x + 1 - 2x + 2 )</td>
</tr>
<tr>
<td></td>
<td>( 2x^2 - 2x = -x + 3 )</td>
</tr>
<tr>
<td></td>
<td>( 2x^2 - 2x + x - 3 = 0 )</td>
</tr>
<tr>
<td></td>
<td>( 2x^2 - x - 3 = 0 )</td>
</tr>
<tr>
<td></td>
<td>( (2x - 3)(x + 1) = 0 )</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Step 3</th>
<th>Determine the roots of the equation obtained using any of the methods. Is the equation factorable? Yes, we can use factoring. Note that you can other methods but factoring is the easiest.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 2x - 3 = 0 )</td>
</tr>
<tr>
<td></td>
<td>( x + 1 = 0 )</td>
</tr>
<tr>
<td></td>
<td>( 2x = 3 )</td>
</tr>
<tr>
<td></td>
<td>( x = -1 )</td>
</tr>
<tr>
<td></td>
<td>( x = \frac{3}{2} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 4</th>
<th>Check if the solutions will make the equation true. Notice that one of the values we obtained is a restriction to the problem. Let’s see by substituting to the original equation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Checking:</td>
<td>for ( x = \frac{3}{2} ) ( \frac{2x}{x+1} = \frac{1}{x-1} - \frac{2}{x+1} ) ( \frac{2 \times \frac{3}{2}}{\frac{3}{2} + 1} = \frac{1}{\frac{3}{2} - 1} - \frac{2}{\frac{3}{2} + 1} ) ( \frac{3 \times 1}{\frac{3}{2} + 1} = \frac{1}{\frac{3}{2} - 1} - \frac{2}{\frac{3}{2} + 1} ) Since the denominator is 0, the equation becomes undefined.</td>
</tr>
<tr>
<td></td>
<td>for ( x = -1 ) ( \frac{2x}{x+1} = \frac{1}{x-1} - \frac{2}{x+1} ) ( \frac{2 \times (-1)}{\frac{3}{2} + 1} = \frac{1}{\frac{3}{2} - 1} - \frac{2}{\frac{3}{2} + 1} ) ( \frac{2 \times (-1)}{\frac{3}{2} + 1} = \frac{1}{\frac{3}{2} - 1} - \frac{2}{\frac{3}{2} + 1} ) Since the denominator is 0, the equation becomes undefined.</td>
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</table>
We have arrived at the values $x = \frac{3}{2}$ and $x = -1$. The equation $\frac{2x}{x+1} = \frac{1}{x-1} - \frac{2}{x+1}$ is true at $x = \frac{3}{2}$, however it is undefined at $x = -1$. Therefore, $x = \frac{3}{2}$ is the only solution to the equation. Henceforth, $x = -1$ is an extraneous root and not a solution to the equation.

An extraneous root or solution is a solution of an equation derived from the original equation but it is not a solution to the original equation. If there are two extraneous roots, then there is no solution to the equation.

**Example #3.** Solve for $x$. \[ \frac{1}{x-3} + \frac{x}{x-2} = \frac{1}{x^2-5x+6} \]

At what values of $x$ will the equation undefined? The expression \[ x^2 - 5x + 6 = (x - 3)(x - 2) \] in factored form. Therefore, $x \neq 3$ and $x \neq 2$.

<table>
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<tr>
<th>Step 1</th>
<th>Remove the denominator by multiplying both sides of the equation by the Least Common Multiple (LCM) of all denominators. What is the LCM in the given equation? Answer: $(x-3)(x-2)$ (Question: How did you find the LCM?)</th>
</tr>
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</tbody>
</table>

One of the values we obtained is $x = 3$. Remember that $x \neq 3$ for the equation to be defined. Can you show why by substitution.
What’s More

Activity 3. Transform then Solve.
Directions: The rational algebraic equations below are reducible to quadratic equations. Reduce the following equations to quadratic equations then find the solutions. Check for extraneous solutions. You can follow the suggested steps in the previous discussion.

1. \(\frac{2}{x+1} + \frac{1}{2x} = \frac{x}{x^2+2x+1}\)

2. \(\frac{a}{a+3} - \frac{8}{a+6} = 0\)

3. \(\frac{x-1}{x-3} - \frac{1}{x} = \frac{2}{x-3}\)

4. \(\frac{3x-2}{x-2} - \frac{6}{(x-2)(x+2)} = 1\)

What I Have Learned

Activity 4. Agree or Disagree
Do you agree or disagree with the given statements below? Defend your answer in at least 2 sentences.
1. Not all rational algebraic equations can be expressed in quadratic equations.
2. Rational algebraic equations that are transformable to quadratic equations have exactly two solutions.

What I Can Do

Activity 5. “Gulayan sa Bakuran Challenge”
Read and understand the situation below, then answer the questions that follow.

The City of Kidapawan invites every household to join the “Magpuyo sa Balay, Mananom ug Gulay” contest, a stay at home initiative project of the said LGU. Maria and his father would like to participate and make their own vegetable garden. Maria thinks that if she do the work alone, it would take her 5 more days than the time her father takes to prepare the plot, cultivate and plant. However, if they work together, they can complete the job in 6 days.

Questions:
1. If Kulas can finish the job in x days, how long will it take Maria to finish the job alone?
2. What mathematical expression would represent the amount of work that Kulas can finish the job in 1 day? How about Maria?
3. What equation would represent the amount of work they can finish in 1 day if they work together?
4. How would you describe the equation formulated in item #3?
5. How will you solve the equation formulated?
6. If you were Maria, would you prefer to work alone or with anyone? Why? What is the importance of teamwork and cooperation?
7. Many youngsters and adults joined social media challenges like Tiktok, Face App challenge and others. Have you tried the “Gulayan sa Bakuran Challenge”? What other challenges did you do together with your family to ensure safety during quarantine and strengthen your bond at the same time? What realizations have you made while experiencing such crisis?

**Assessment**

**Summative Test.**

Directions: Let us check how much you have learned today. Read each question carefully. Encircle the letter of the correct answer.

1. The following equations can be transformed to quadratic equations EXCEPT
   A. \((x - 1)^2 + (x - 4)^2 = 20\)  
   B. \(x^2 - 5 = x^2 + 2x + 5\)  
   C. \((x - 3)(x + 7) = 0\)  
   D. \((x + 1)^2 - 4 = 9\)

2. The equation \(x(x - 5) = 4x\) can be solved using which method/s?
   I. Extracting square root  
   II. Factoring  
   III. Completing the square  
   IV. Quadratic formula  
   A. II only  
   B. IV only  
   C. II, III and IV  
   D. I, II, III, IV

3. What are the roots of the equation \(x(x + 10) + 20 = -5\)?
   A. \(-5, -5\)  
   B. \(5, 5\)  
   C. \(10, 2\)  
   D. \(-10, -2\)

4. One of the roots of the equation \((x + 2)^2 = 2(x + 6)\) is 2. What is the other root?
   A. \(-2\)  
   B. 2  
   C. \(-4\)  
   D. 4

5. Which of the quadratic equations has equal roots?
   A. \((x + 8)(x + 5) = x + 4\)  
   B. \((x - 4)^2 - 9 = 0\)  
   C. \(x^2 - 8 = 4\)  
   D. \((x + 1)^2 + (x - 3)^2 = 1\)

6. What quadratic equation can be solved using factoring?
   A. \((x - 4)^2 - 9 = 0\)  
   B. \(x(2x + 5) = 8\)  
   C. \(x^2 + 3x = -6\)  
   D. \((x + 1)^2 + (x - 3)^2 = 16\)

7. The roots of a quadratic equation are -3 and -5. What quadratic equation has these roots?
   A. \((x + 8)^2 = -15\)  
   B. \(x(x - 8) = -15\)  
   C. \(x^2 = x + 15\)  
   D. \(x^2 + 8x = 15\)

8. How many real roots does the equation \(-(x + 2)(x - 1) = 3\) have?
   A. 0  
   B. 1  
   C. 2  
   D. 3
9. Which of the following statements is true about rational algebraic equations?
A. There are rational algebraic equations that cannot be reduced into quadratic equations.
B. All values obtained from solving a rational algebraic equations are solutions to the said equation.
C. The equation \( \frac{2}{2x} - \frac{1}{x} = \frac{1}{3} \) is an example of a rational algebraic equation transformable to quadratic equations.
D. All rational algebraic equations have exactly two real solutions.

10. What should be multiplied to remove the denominators of the equation \( \frac{1}{x} - \frac{2}{x+1} = \frac{3}{4} \)?
A. 4x(x + 1)  
B. 4(x + 1)  
C. x(x + 1)  
D. 12x(x + 1)

11. Which quadratic equation is equivalent to \( \frac{1}{3} - \frac{2}{x+1} = \frac{3}{4} \)?
A. \(3x^2 - 4x + 7 = 0\)  
B. \(3x^2 + 7x - 4 = 0\)  
C. \(4x^2 - 3x - 7 = 0\)  
D. \(4x^2 + 7x - 3 = 0\)

12. Which of the rational algebraic equations has extraneous roots?
A. \(\frac{2}{c-1} + \frac{3c}{c+1} = 4\)  
B. \(\frac{9x}{x-3} - \frac{3}{x(x-3)} = 0\)  
C. \(\frac{a}{a^2 - 2} = \frac{-1}{a}\)  
D. \(\frac{6y}{3y+1} = \frac{1}{y} - \frac{2}{3y+1}\)

From items 13-15, refer to the equation \(\frac{x+1}{x^2} + \frac{1}{2x} = \frac{1}{4}\).

13. What value/s of x will make the equation undefined?
A. 0  
B. 0, 2, 4  
C. 0, 2  
D. 2

14. When the equation is written in standard form, it will be equal to
A. \(x^2 + 6x - 4 = 0\)  
B. \(x^2 + 4x - 6 = 0\)  
C. \(x^2 - 6x - 4 = 0\)  
D. \(x^2 + 4x - 6 = 0\)

15. What are the roots of the equation?
A. 3, 3  
B. \(-\sqrt{3}, \sqrt{3}\)  
C. \(3 - \sqrt{3}, 3 + \sqrt{3}\)  
D. \(-3, 3\)

**Additional Activities**

**Think of these . . .**

A. Solve the equation \(4(2x^2 - 3) = 3x^2 - 7x - 12\) using all possible methods. Which of the methods are difficult to use? Which is the easiest? Explain.

B. Your classmate solve the equation \(\frac{3x^2 - 6}{8-x} = x - 2\) and found no solutions to the equation. Do you agree with him? Justify your answer.
Answer Key

What I Know

1. B
2. C
3. A
4. A
5. C
6. D
7. C
8. A
9. A
10. B
11. A
12. D
13. C
14. A
15. B

What's In (L1)

QE that can be solved using factoring
1. $x^2 + 8x + 16 = 0$
2. $x^2 - 6x - 36 = 0$
3. $x^2 + 18x + 75 = 0$
4. $3x^2 + 14x - 5 = 0$
5. $4x^2 + 9 = 0$
6. $x^2 + 16x + 30 = 0$
7. $x^2 - 2x = 0$
8. $x^2 + 2x + 1 = 0$

QE that cannot be solved using factoring
1. $x^2 - 4x + 1 = 0$
2. $x^2 + 3x - 1 = 0$
3. $x^2 - 4x + 6 = 0$
4. $x^2 - 4x + 9 = 0$

What's More (L1)

1. (L1) $c + (d + c) = 240$
2. $x^2 + 4x - 45 = 0$
3. $x = 5, x = -9$ (reject -9 since the values of the dimensions are all positive)
4. $w = 5cm, h = 6cm$

1. (L2) $\sqrt{x} + \frac{1}{x} = 3$
2. $x^2 + 2x + 1 = 0$
3. $x^2 - 4x + 6 = 0$
4. $x^2 - 16x + 6 = 0$

WHAT I CAN DO

1. $c + (d + c) = 240$
2. $x^2 + 4x - 45 = 0$
3. $x = 5, x = -9$ (reject -9 since the values of the dimensions are all positive)
4. $w = 5cm, h = 6cm$

Assessment

1. B
2. C
3. A
4. A
5. B
6. D
7. B
8. A
9. A
10. B
11. D
12. A
13. C
14. C
15. B

(L2)

1. $\frac{3}{x} + \frac{6}{x} = \frac{1}{x}$
2. $\frac{1}{x} + \frac{1}{x} = \frac{1}{x}$
3. $\frac{1}{x} + \frac{1}{x} = \frac{1}{x}$
4. $\frac{1}{x} + \frac{1}{x} = \frac{1}{x}$

1. Kulas: $\frac{1}{x}$
2. Maria: $\frac{1}{x}$
3. Kulas: $\frac{1}{x}$
4. Maria: $\frac{1}{x}$
5. The solution is 10 days for Kulas and 15 days for Maria.
**References**

Deped Grade 9 Teaching Guide Module 1 Lesson 5: Equations Transformable to Quadratic Equations

Deped Learner's Material for Mathematics Grade 9 Module 1 Lesson 5: Equations Transformable to Quadratic Equations

EASE Module 3: Quadratic Equations


http://www.analyzemath.com/Algebra2/Algebra2.html

http://www.analyzemath.com/Algebra2/algebra2_solutions.html
DISCLAIMER

This Self-learning Module (SLM) was developed by DepEd SOCCSKSARGEN with the primary objective of preparing for and addressing the new normal. Contents of this module were based on DepEd’s Most Essential Learning Competencies (MELC). This is a supplementary material to be used by all learners of Region XII in all public schools beginning SY 2020-2021. The process of LR development was observed in the production of this module. This is version 1.0. We highly encourage feedback, comments, and recommendations.

For inquiries or feedback, please write or call:

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